A minimal model of insulin secretion and kinetics to assess hepatic insulin extraction

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The liver plays a major role in determining circulating insulin levels because it is responsible for the elimination of the prevalent fraction of insulin. A quantitative assessment of hepatic insulin extraction (HE) in an individual, in the basal state and/or during a glucose perturbation, is an essential step in deriving an overall parametric picture of the glucose regulatory system. Because a direct measurement requires invasive protocols, with catheters placed in the artery and hepatic vein, indirect approaches based on mathematical models are essential to infer HE from plasma measurements (8). Several approaches have been proposed that combine the assessment of insulin secretion (ISR) from C-peptide data and insulin delivery rate (IDR) in plasma after its first pass through the liver from insulin data to provide HE noninvasively in humans during a glucose perturbation, either intravenously or orally (6, 7, 22, 24). Linking ISR and IDR to plasma measurements of C-peptide and insulin requires a description of C-peptide and insulin kinetics. Different compartmental structures were assumed for insulin (from 1 to 3 compartments) and C-peptide kinetics (1 or 2 compartments), as well as different functional descriptions for insulin secretion/delivery during glucose perturbation. Because all these approaches were aimed at simultaneously estimating kinetics and secretion, assessment of secretion could be biased because of undesired compensations that may arise in parameter estimation of kinetics and secretion processes (20). Moreover, HE was often assumed to be constant during the experiment, even if evidence exists that this process varies in time during a glucose/insulin perturbation (6, 15), e.g., due to saturable receptor-mediated mechanisms (12) and/or dependence of HE from blood flow (22).

The purpose here is to assess HE during an intravenous glucose load in humans by using the minimal-modeling approach. This approach has already been applied to C-peptide data, measured during an intravenous glucose tolerance test (IVGTT) (20) to develop the minimal model of C-peptide secretion and kinetics and then extended to an insulin-modified (IM) IVGTT; i.e., a bolus or a 5-min square wave of insulin is infused 20 min after the glucose bolus (21). Standard population values were used for C-peptide kinetics (23) to avoid compensation between kinetic and secretion parameters, thus reliably measuring insulin secretion and indexes of β-cell responsivity to glucose. Unfortunately, the extension of this approach to assess insulin delivery rate and kinetics from insulin data is not straightforward, mainly because standard values are not available for insulin kinetics. To solve this problem, we took advantage of the unique features of IM-IVGTT: these data are more informative than standard IVGTT because the decay of insulin concentration observed after the exogenous insulin administration facilitates a reliable estimation of insulin kinetics in each individual, a prerequisite to avoiding errors in modeling secretion. The combined use of insulin and C-peptide minimal models was then explored as a tool to assess HE.

Database. The data originated from a previous study (2), to which we refer for details on experimental protocols and analytical methods. Briefly, 20 normal subjects (16 males and 4 females, mean age 44 ± 5 yr, BMI 26.03 ± 0.68 kg/m²) underwent an IM-IVGTT that consisted of an intravenous injection of glucose (0.3 g/kg body wt) followed by a square wave (from 20 to 25 min) of insulin infusion (0.02 U/kg body wt). Blood was sampled at 0, 2, 4, 6, 8, 10, 15, 20, 22, 25, 26, 28, 31, 35, 45, 60, 75, 90, 120, 180, and 240 min after the glucose injection, and glucose, C-peptide, and insulin concentrations were measured.

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METHODS

C-peptide model. The minimal model of C-peptide secretion and kinetics that we described earlier (21) was used to assess ISR. C-peptide kinetics are described by the linear two-compartment model (Fig. 1) that was originally proposed by Eaton et al. (7)

\[
\begin{align*}
CP_1(t) &= -[k_{01} + k_{21}]CP_1(t) + k_{12}CP_2(t) + \text{isr}(t) \quad CP_1(0) = 0 \\
CP_2(t) &= k_{21}CP_1(t) - k_{12}CP_2(t) \quad CP_2(0) = 0
\end{align*}
\]

(1)

where \( CP_1 \) and \( CP_2 \) (pmol/l) are C-peptide concentrations (above basal) in the accessible and peripheral compartments, respectively; \( k_{01}, k_{21}, \) and \( k_{12} \), transfer rate parameters; \( \text{isr} \), insulin secretion above basal; \( \dot{Y} \), provision of new insulin to the \( \beta \)-cells controlled by glucose concentration; \( X_0 \), amount of insulin secreted as an impulse during first-phase secretion in response to rapid increase of glucose concentration after glucose bolus.

It was assumed that \( \text{isr} \) was proportional to the amount \( X \) (pmol/l) of insulin in the \( \beta \)-cells, resulting from the balance between \( \text{isr} \) and provision \( Y \) (pmol/l) of new insulin to the \( \beta \)-cells.

\[
\begin{align*}
\text{isr}(t) &= mX(t) \\
X(t) &= -mX(t) + Y(t) \quad X(0) = X_0
\end{align*}
\]

(2)

\( X_0 \) is responsible for first-phase secretion, whereas the slower second phase derives from provision \( Y \), which is controlled by glucose according to the following equation:

\[
Y(t) = -\alpha[Y(t) - \beta(G(t) - h)] \quad Y(0) = 0
\]

(4)

i.e., in response to an elevated glucose level, \( Y \) and thus \( \text{isr} \) tends with a time constant of \( 1/\alpha \) (min) toward a steady-state value that is linearly related via parameter \( \beta \) (min \(^{-1}\)) to glucose concentration \( G \) (mmol/l) above the threshold value \( h \).

From model parameters, the \( \text{ISR} \) profile (pmol/min) was reconstructed as

\[
\text{ISR}(t) = [k_{01}C_{ib} + \text{isr}(t)]V_C = [k_{01}C_{ib} + mX(t)]V_C
\]

(5)

where \( V_C \) is the distribution volume of the accessible compartment, and \( C_{ib} \) is the end-test C-peptide concentration.

Indices of responsivity to glucose of first- and second-phase and basal secretion were also calculated. First-phase responsivity to glucose, \( \Phi_1 \) (dimensionless), is defined as the ratio between the incremental amount of insulin secreted during the first phase and the maximum increment of plasma glucose concentration \( \Delta G \) (mmol/l).

\[
\Phi_1 = X_0/\Delta G
\]

(6)

Second-phase responsivity to glucose, \( \Phi_2 \) (min \(^{-1}\)) equals parameter \( \beta \), which describes the stimulatory effect of glucose concentration on provision

\[
\Phi_2 = \beta
\]

(7)

Finally, basal responsivity to glucose, \( \Phi_b \) (min \(^{-1}\)), is defined as

\[
\Phi_b = \text{isr}/G_b = k_{01}C_{ib}/G_b
\]

(8)

where \( G_b \) is the end-test glucose concentration.

Insulin model. To assess IDR from plasma insulin concentration, a linear single-compartment model was used (Fig. 2):

\[
I(t) = -nI(t) + \text{idr}(t) + U(t)/V_I \quad I(0) = 0
\]

(9)

where \( I \) (pmol/l) is insulin concentration above basal (end-test) value, \( n \) (min \(^{-1}\)) is the rate constant of insulin disappearance, \( \text{idr} \) (pmol/l/min \(^{-1}\)) is insulin delivery rate above basal normalized to the volume of distribution, \( U \) (pmol/min) is the exogenous insulin input, different from zero in the 20–25-min interval, and \( V_I \) (l) is the insulin distribution volume.

For \( \text{idr} \), a description similar to that proposed to describe insulin secretion from C-peptide data was adopted
idr(t) = m_{IDR} X_{IDR}(t) \\
X_{IDR}(t) = - m_{IDR} X_{IDR}(t) + Y_{IDR}(t) \\
Y_{IDR}(t) = - \alpha_{IDR} [Y_{IDR}(t) - \beta_{IDR} [G(t) - h]] \\
Y_{IDR}(0) = 0 \tag{10}

where secretory parameters and variables are labeled “idr” to indicate that they refer to insulin delivery rate and then account for secretion and hepatic extraction.

From model parameters, the IDR profile (pmol/min) was reconstructed as

\[ IDR(t) = [n_b + idr(t)] V_I = \left[ n_b + m_{IDR} X_{IDR}(t) \right] V_I \tag{11}\]

where \( n_b \) is the end-test insulin concentration.

Indexes of responsivity to glucose of first- and second-phase and basal insulin delivery rate were also calculated by using formulas similar to Eqs. 6-8:

\[ \Phi_{IDR}^1 = X_{IDR}^{1/\Delta G} \tag{12} \]
\[ \Phi_{IDR}^2 = \beta_{IDR} \tag{13} \]
\[ \Phi_{IDR}^h = \text{idr}_h / \text{G}_b = \frac{n_b}{g_b} \tag{14} \]

where \( G_b \) is the end-test glucose concentration.

**Hepatic insulin extraction.** From ISR and IDR profiles, the HE profile was reconstructed as

\[ HE(t) = \frac{\text{ISR}(t) - \text{IDR}(t)}{\text{ISR}(t)} = 1 - \frac{\text{IDR}(t)}{\text{ISR}(t)} \tag{15} \]

From ISR and IDR indexes, two HE indexes, i.e., during IVGTT and in the basal state, were estimated as

\[ HE = \frac{\int_0^T \text{ISR}(t) dt - \int_0^T \text{IDR}(t) dt}{\int_0^T \text{ISR}(t) dt} = 1 - \frac{(\Phi_{IDR} + \Phi_1 A_1 + \Phi_2 A_2) V_I}{(\Phi_b + \Phi_1 A_1 + \Phi_2 A_2) V_C} \]

\[ HE_b = \frac{\text{ISR}_b \cdot V_C - \text{IDR}_b \cdot V_I}{\text{ISR}_b \cdot V_C} = 1 - \frac{\Phi_{IDR}^h V_I}{\Phi_b V_C} \tag{16} \]

\[ A_1 = \frac{\Delta G}{T \cdot G_h} \quad A_2 = \frac{\int [G(t) - h] dt}{T \cdot G_h} \tag{17} \]

where \( T \) is the time at which glucose, insulin, and C-peptide concentrations reach their end-test values after the IVGTT perturbation.

**Model identification.** Identification of the two models was performed simultaneously in each subject by using SAAM II software (1), with the constraint that the threshold, \( h \), assumes the same value in both models. Secretion parameters \( m, \alpha, \beta, \) and \( X_0 \) were estimated from plasma C-peptide, and glucose data with C-peptide kinetic parameters were fixed to standard population values, following the method proposed by Van Cauter et al. (23), whereas insulin kinetic \( (n \) and \( V_I) \) and delivery rate parameters \( (m_{IDR}, \alpha_{IDR}, \beta_{IDR}, X_0^{IDR}) \) were estimated from plasma insulin and glucose data.

Errors in insulin and C-peptide measurements were assumed to be uncorrelated, Gaussian, zero mean with a variance linked to insulin (1), and C-peptide (C) measurements according to the models, \( \text{Var}(I) = 6 + 0.0055 \times F \) and \( \text{Var}(C) = 2000 + 0.001 \times C^2 \). The coefficients of variations (CVs) of insulin measurements then ranged from \( \sim 13\% \) at basal level down to \( 8\% \) (16) and that of C-peptide from \( 11\% \) down to \( 4\% \).

**RESULTS**

Average glucose, insulin, and C-peptide concentrations are shown in Fig. 3. A steady-state condition was reached at the end of the experiment at levels (4.58 ± 0.24 mmol/l, 18.3 ± 6.3 pmol/l, and 403 ± 114 pmol/l, respectively) somewhat lower than the pretest levels (5 ± 0.33 mmol/l, 25.3 ± 6.5 pmol/l, and 471 ± 121 pmol/l, respectively).

The ability of C-peptide and insulin minimal models to describe the data is shown in Fig. 4, where average weighted residuals are shown.

Estimated secretion parameters \( m, \alpha, \beta, X_0, \) and \( h \) and indexes \( \Phi_b, \Phi_1, \) and \( \Phi_2 \) are shown in Table 1. Estimated insulin kinetic parameters \( n \) and \( V_I \), delivery rate parameters \( n_{IDR}, \alpha_{IDR}, \beta_{IDR}, \) and \( X_0^{IDR} \), and indexes \( \Phi_{IDR}^1, \Phi_1^{IDR}, \) and \( \Phi_2^{IDR} \) in all individuals are summarized in Table 2.
Parameter \( h \) was, on average, \( 4.6 \pm 0.3 \text{ mmol/l} \), not significantly different from the end-test value of glucose concentration.

The minimal model-derived average HE profile is shown in Fig. 5. Starting from a pretest value of \( 64 \pm 12\% \), HE is rapidly suppressed after the glucose bolus and then slowly reaches, within 120 min, a steady-state value \( \text{HE}_{b} \) of \( 70 \pm 9\% \). Individual values of \( \text{HE}_{b} \) and HE indexes are shown in Table 3 and indicate that \( \sim 50\% \) of secreted insulin is extracted by the liver during an IM-IVGTT.

### DISCUSSION

The minimal-modeling approach, originally proposed to assess ISR during an IVGTT on the basis of C-peptide and glucose data, has been applied here to quantify IDR from insulin data, thus making it possible to noninvasively assess HE. The adopted model uses a functional description of the control of glucose on IDR similar to that proposed by us (20) for ISR. The model also includes a description of insulin kinetics to link secretion to insulin measurements taken in plasma. To reliably assess insulin kinetics, a prerequisite to avoid undesired compensations between parameters describing secretion and kinetics (20), the insulin minimal model was identified on IM-IVGTT data. This protocol associates the glucose bolus with a short insulin infusion 20 min after the bolus and was originally proposed to improve the identification of the glucose minimal model (9, 25). It also offers definite advantages for the identification of the insulin minimal model, because insulin decay data, measured after the insulin infusion, allowed a reliable estimation of insulin disappearance rate constant and distribution volume and, thus, of insulin clearance rate in each individual. Secretory parameters were estimated as well, thus allowing us to reconstruct the posthepatic secretion profile as well as derive indexes of first- and second-phase posthepatic secretion.

The combined use of insulin and C-peptide minimal models identified in the same subject provided quantitative information on the ability of the liver to extract insulin during any passage through it, because it enabled reconstruction of HE profile and calculation of two indexes that were able to quantify this process both in the basal state and during the glucose perturbation.

**Insulin kinetics.** A variety of models have been used in the literature to describe insulin kinetics, with the number of compartments ranging from one to three. Moreover, it has been suggested that insulin clearance may vary during a glucose and/or insulin perturbation. This is invariably reported in the case of sustained perturbations such as, e.g., 3-h hyperglycemic glucose clamp in humans (19), 1-h insulin infusion in dogs (15), and 42-h low-rate glucose infusion in humans (4). In the case of transient perturbations, such as those induced by an IM-IVGTT, it is generally acknowledged that insulin kinetics can be considered linear until insulin levels of 100–150 μU/ml (i.e., 600–900 pmol/l) are reached (5, 10, 12, 15). In the present study, a linear single-compartment model was assumed to describe insulin kinetics. The linearity assumption appears reasonable because insulin concentration remains within the linearity range for the largest part of the experiment. The monocompartmental assumption renders the whole model (secretion plus kinetics) uniquely a priori identifiable, which allowed us to estimate kinetic parameters with acceptable precision. Moreover, it provided a reasonable fit, particularly of data collected immediately after the exogenous insulin administration, and reliable estimates of kinetic parameters; e.g., insulin plasma clearance rate (PCR = \( n \cdot V_{l} \)) was \( 1.7 \pm 0.36 \) (means ± SD) l/min, very similar to values reported by some authors (4, 17, 19). These findings confirm that the linear single-compartment model provides a reasonable description of insulin kinetics in situations like the present one. They also

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**Table 1. Parameters and indexes of C-peptide model**

<table>
<thead>
<tr>
<th>Subject</th>
<th>( m_{s} )</th>
<th>( \alpha_{s} )</th>
<th>( X_{b} )</th>
<th>( \Phi_{1} )</th>
<th>( \Phi_{2} )</th>
<th>( \Phi_{3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.90 (71)</td>
<td>0.17 (26)</td>
<td>2.429 (4)</td>
<td>182 (4)</td>
<td>10.1 (4)</td>
<td>6.8</td>
</tr>
<tr>
<td>2</td>
<td>0.75 (42)</td>
<td>0.13 (28)</td>
<td>1.766 (4)</td>
<td>147 (4)</td>
<td>6.7 (7)</td>
<td>2.8</td>
</tr>
<tr>
<td>3</td>
<td>0.43 (56)</td>
<td>0.05 (19)</td>
<td>1.094 (5)</td>
<td>72 (5)</td>
<td>6.7 (10)</td>
<td>4.8</td>
</tr>
<tr>
<td>4</td>
<td>0.87 (48)</td>
<td>0.05 (17)</td>
<td>2.589 (4)</td>
<td>173 (4)</td>
<td>7.6 (8)</td>
<td>7.6</td>
</tr>
<tr>
<td>5</td>
<td>0.61 (32)</td>
<td>0.07 (14)</td>
<td>1.093 (6)</td>
<td>55 (6)</td>
<td>7.8 (6)</td>
<td>4.2</td>
</tr>
<tr>
<td>6</td>
<td>0.56 (68)</td>
<td>0.05 (20)</td>
<td>2.556 (9)</td>
<td>41 (9)</td>
<td>5.8 (10)</td>
<td>4.7</td>
</tr>
<tr>
<td>7</td>
<td>0.39 (19)</td>
<td>0.11 (22)</td>
<td>2.101 (5)</td>
<td>156 (5)</td>
<td>7.7 (6)</td>
<td>5.9</td>
</tr>
<tr>
<td>8</td>
<td>0.66 (32)</td>
<td>0.10 (16)</td>
<td>1.251 (6)</td>
<td>83 (6)</td>
<td>10.0 (5)</td>
<td>8.0</td>
</tr>
<tr>
<td>9</td>
<td>0.38 (34)</td>
<td>0.19 (19)</td>
<td>1.085 (6)</td>
<td>84 (6)</td>
<td>11.2 (4)</td>
<td>5.0</td>
</tr>
<tr>
<td>10</td>
<td>0.34 (54)</td>
<td>0.02 (21)</td>
<td>1.870 (4)</td>
<td>193 (4)</td>
<td>13.5 (15)</td>
<td>4.5</td>
</tr>
<tr>
<td>11</td>
<td>0.16 (16)</td>
<td>0.13 (37)</td>
<td>2.518 (8)</td>
<td>305 (8)</td>
<td>6.9 (12)</td>
<td>5.7</td>
</tr>
<tr>
<td>12</td>
<td>0.64 (39)</td>
<td>0.19 (24)</td>
<td>2.067 (5)</td>
<td>184 (5)</td>
<td>10.7 (5)</td>
<td>5.2</td>
</tr>
<tr>
<td>13</td>
<td>0.71 (53)</td>
<td>0.13 (18)</td>
<td>1.321 (5)</td>
<td>106 (5)</td>
<td>9.7 (4)</td>
<td>5.8</td>
</tr>
<tr>
<td>14</td>
<td>0.92 (18)</td>
<td>0.12 (35)</td>
<td>1.119 (5)</td>
<td>65 (5)</td>
<td>4.5 (8)</td>
<td>4.6</td>
</tr>
<tr>
<td>15</td>
<td>0.77 (28)</td>
<td>0.18 (37)</td>
<td>1.372 (4)</td>
<td>161 (4)</td>
<td>6.5 (8)</td>
<td>3.8</td>
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<tr>
<td>16</td>
<td>0.78 (47)</td>
<td>0.16 (29)</td>
<td>1.991 (4)</td>
<td>168 (4)</td>
<td>7.2 (7)</td>
<td>6.1</td>
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<tr>
<td>17</td>
<td>1.13 (41)</td>
<td>0.17 (17)</td>
<td>4.302 (4)</td>
<td>277 (4)</td>
<td>15.1 (4)</td>
<td>7.2</td>
</tr>
<tr>
<td>18</td>
<td>0.70 (47)</td>
<td>0.27 (62)</td>
<td>1.440 (5)</td>
<td>126 (5)</td>
<td>4.5 (8)</td>
<td>2.8</td>
</tr>
<tr>
<td>19</td>
<td>0.76 (23)</td>
<td>0.16 (25)</td>
<td>2.724 (3)</td>
<td>205 (3)</td>
<td>3.3 (11)</td>
<td>2.9</td>
</tr>
<tr>
<td>20</td>
<td>0.53 (28)</td>
<td>0.36 (63)</td>
<td>2.563 (5)</td>
<td>260 (5)</td>
<td>9.5 (7)</td>
<td>5.3</td>
</tr>
<tr>
<td>Mean</td>
<td>0.66 (38)</td>
<td>0.14 (27)</td>
<td>1.863 (5)</td>
<td>152 (5)</td>
<td>8.2 (7)</td>
<td>5.2</td>
</tr>
</tbody>
</table>

SD 0.22 0.08 850 75 3.0 1.5

*Precision of parameter estimate expressed as %coefficient of variation. See text for definitions of parameters and indexes.
support the validity of the minimal-model description of IDR, because we showed (20) that a reliable description of kinetic processes avoids errors in modeling the secretory processes.

**IDR.** The model adopts a functional description of the control of glucose on IDR similar to that used in the C-peptide minimal model (20). Parameters and indexes estimated from insulin data (Table 2) assumed values that were different from their counterparts estimated from C-peptide data (Table 1), because they also accounted for HE. Only the threshold of glucose concentration that is able to stimulate ISR and IDR is forced to be the same. The estimated value is very close to the constant level reached by glucose at the end of the experiment, when insulin and C-peptide concentrations are almost constant.

These concentrations were adopted as the basal concentrations 9% different from the pretest value (64% 9%), which was forced to be the same. The estimated value is very close to the constant level reached by glucose at the end of the experiment, when insulin and C-peptide concentrations are almost constant.

First- and second-phase responsivity to glucose were 57 ± 28·10⁻⁹·min⁻¹ and 2.2 ± 0.9·10⁻⁹·min⁻¹, accounting for 39 and 30%, respectively (77 and 59%, respectively, after volume correction), of their pancreatic counterparts.

**HE.** HE was assessed by combining the information derived from the two minimal models. The HE profile showed a rapid suppression immediately after the glucose bolus administration, followed by a rapid increase that was attenuated at 20 min, when insulin was administered (Fig. 5). Then, it returned to steady state at the end of the experiment at a value (70 ± 9%) different from the pretest value (64 ± 12%), which was

### Table 2. Parameters and indexes of insulin model

<table>
<thead>
<tr>
<th>Subject</th>
<th>( n, \text{ min}^{-1} )</th>
<th>( Y_0, \text{ pmol} )</th>
<th>( \alpha_{IDR}, \text{ min}^{-1} )</th>
<th>( X_{IDR}, \text{ pmol} )</th>
<th>( \Phi_{IDR}, 10^{-9} )</th>
<th>( \Phi_{IDR}, 10^{-9} \cdot \text{min}^{-1} )</th>
<th>( \Phi_{IDR}, 10^{-9} \cdot \text{min}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.18 (8)*</td>
<td>9.7 (8)</td>
<td>0.51 (30)</td>
<td>0.16 (30)</td>
<td>912 (7)</td>
<td>68 (7)</td>
<td>2.9 (14)</td>
</tr>
<tr>
<td>2</td>
<td>0.27 (6)</td>
<td>6.8 (7)</td>
<td>1.11 (92)</td>
<td>0.08 (12)</td>
<td>875 (20)</td>
<td>73 (20)</td>
<td>3.2 (11)</td>
</tr>
<tr>
<td>3</td>
<td>0.12 (6)</td>
<td>13.1 (6)</td>
<td>0.59 (28)</td>
<td>0.01 (38)</td>
<td>382 (6)</td>
<td>25 (6)</td>
<td>0.9 (22)</td>
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<td>0.14 (8)</td>
<td>11.2 (6)</td>
<td>0.88 (26)</td>
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<td>834 (8)</td>
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<td>0.83 (27)</td>
<td>0.05 (12)</td>
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<td>12 (8)</td>
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<td>9.8 (7)</td>
<td>0.78 (28)</td>
<td>0.04 (12)</td>
<td>243 (8)</td>
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<td>1.02 (85)</td>
<td>0.15 (21)</td>
<td>1060 (14)</td>
<td>79 (14)</td>
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<td>0.25 (9)</td>
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<td>0.84 (27)</td>
<td>0.08 (11)</td>
<td>484 (11)</td>
<td>32 (11)</td>
<td>4.0 (13)</td>
</tr>
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<td>7.9 (8)</td>
<td>0.17 (30)</td>
<td>0.17 (21)</td>
<td>281 (16)</td>
<td>22 (16)</td>
<td>1.9 (20)</td>
</tr>
<tr>
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<td>7.6 (9)</td>
<td>0.66 (31)</td>
<td>0.02 (23)</td>
<td>538 (7)</td>
<td>55 (7)</td>
<td>3.1 (15)</td>
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<td>0.17 (11)</td>
<td>9.1 (11)</td>
<td>0.31 (24)</td>
<td>0.29 (65)</td>
<td>585 (10)</td>
<td>71 (10)</td>
<td>1.7 (28)</td>
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<tr>
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<td>12.0 (6)</td>
<td>0.70 (25)</td>
<td>0.03 (16)</td>
<td>467 (7)</td>
<td>42 (7)</td>
<td>1.9 (16)</td>
</tr>
<tr>
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<td>9.4 (8)</td>
<td>0.81 (27)</td>
<td>0.08 (11)</td>
<td>505 (9)</td>
<td>40 (9)</td>
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<td>14</td>
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<td>2.4 (18)</td>
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<td>2.1 (19)</td>
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<tr>
<td>Mean</td>
<td>0.19 (8)</td>
<td>9.2 (8)</td>
<td>0.64 (31)</td>
<td>0.09 (23)</td>
<td>701 (9)</td>
<td>57 (9)</td>
<td>2.2 (18)</td>
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<td>SD</td>
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<td>2.2</td>
<td>0.27</td>
<td>0.07</td>
<td>334</td>
<td>28</td>
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*Precision of parameter estimate expressed as %coefficient of variation. See text for definitions of parameters and indexes; IDR, insulin delivery rate.

![Fig. 5. Hepatic extraction during an insulin-modified intravenous glucose tolerance test (mean in 20 normal subjects).](http://ajpendo.physiology.org/)

First- and second-phase responsivity to glucose were 57 ± 28·10⁻⁹·min⁻¹ and 2.2 ± 0.9·10⁻⁹·min⁻¹, accounting for 39 and 30%, respectively (77 and 59%, respectively, after volume correction), of their pancreatic counterparts.

**HE.** HE was assessed by combining the information derived from the two minimal models. The HE profile showed a rapid suppression immediately after the glucose bolus administration, followed by a rapid increase that was attenuated at 20 min, when insulin was administered (Fig. 5). Then, it returned to steady state at the end of the experiment at a value (70 ± 9%) different from the pretest value (64 ± 12%), which was
expected because insulin and C-peptide data are different in the two situations. These results indicate that HE varies in time during a transient glucose perturbation. They are consistent with previous observations, obtained, e.g., in dogs by using invasive methods (13) or in humans by using a modeling approach similar to the one adopted here (6). However, in evaluating the model-predicted pattern of HE, it is important to consider that its ingredients, the model-predicted ISR and IDR, are in fact the rate of appearance in plasma of C-peptide molecules resulting from secretion and transit through the liver and of newly secreted insulin molecules resulting from secretion, HE, and transit through the liver, respectively. However, the liver not only degrades a fraction of insulin molecules but also introduces a delay; e.g., for insulin, a transit time equal to 43 s in humans (8) and 2 min in dogs (7) was reported. Should the transit time of C-peptide be different than that of insulin, the HE profile shown in Fig. 5 accounts not only for the time course of HE but also for differences in the way the liver handles insulin and C-peptide molecules. By contrast, these differences do not affect HE indexes: HEb is calculated in steady-state conditions, i.e., in a situation where inward and outward hepatic fluxes are equal, whereas HE (Eq. 10) is defined from IDR and ISR integrals that are not affected by the liver delay. Our results (Table 3) indicate that the liver is able to extract 70 ± 9% of the insulin passing through it in the end-test basal state and 54 ± 14% during IM-IVGTT. A possible explanation is that liver saturation mechanisms are activated at insulin concentration levels observed during IM-IVGTT. In addition, one can also call for an increase in hepatic blood flow, induced by IM-IVGTT, which has been shown to be associated with a reduced HE (22). Our values are in good agreement with previous findings in the steady state, as reviewed by Ferrannini and Cobelli (8), and during an IVGTT perturbation (6).

Regarding HE estimation, our results indicate that HEb and HE can be estimated with very good precision, i.e., with CVs respectively equal to 3 and 7%. CVs were calculated by propagating the precision of all model parameters included in HEb and HE formulas (Eqs. 15 and 16). However, glucose data, as well as basal insulin and C-peptide levels, are assumed to be noise free, and C-peptide distribution volume Vc was fixed to a population value and adjusted according to the body surface area, following the method proposed by Van Cauter et al. (23). Because Vc plays a crucial role in HEb and HE calculations, we evaluated through a sensitivity analysis that the average percentage sensitivities of HEb and HE to percentage errors in Vc are 0.45 ± 0.23 and 1.02 ± 0.87, respectively. The order of magnitude of errors in Vc can be evaluated as 20% from a study on seven subjects for whom reference values were available (11). From the sensitivity analysis, they reflected a 10 and 20% increase in HEb and HE precision, thus moving from 3 and 7% (Table 3) to still acceptable values of 15 and 27%, respectively.

**HE and insulin clearance.** HE is a component of insulin PCR, accounting for ~58% of the total flux (from the data reviewed in Ref. 8). Our data in fact indicate a significant correlation of PCR with both HEb (R = 0.48, P = 0.03) and HE (R = 0.60, P = 0.005).

The reduction predicted by the two minimal models in HE during IM-IVGTT is apparently in contrast with the linearity assumption of insulin kinetics. However, the 23% reduction in HE estimated during an IM-IVGTT with respect to basal (from 70 to 54%; Table 3), although reflecting in a 23% reduction of the hepatic component of insulin clearance (from 0.58 PCR to 0.45 PCR), accounts only for a lower percentage decrease of insulin clearance (to 0.45 PCR + 0.42 PCR = 0.87 PCR) if nonhepatic insulin clearance is assumed to be linear during IM-IVGTT (thus remaining constant at 0.42 PCR). Because a 13% reduction of insulin clearance is hard to detect from our data, we can speculate that insulin clearance is likely to vary in time during an IM-IVGTT due to variations in HE, but these variations are modest and difficult to detect from plasma insulin data. Only a constant value of PCR can be estimated, which represents the overall clearance value during IM-IVGTT. To check the reliability of these assumptions, we have developed an alternative model that links insulin clearance to HE. The model, detailed in the Appendix, describes insulin clearance as the sum of two processes: hepatic clearance, varying in time with a pattern proportional to HE profile, and peripheral clearance, assumed to be constant. Numerical identification was successfully achieved only by fixing a priori the ratio between the two processes at basal state. This is a critical issue that is likely to vary among subjects/conditions. When a plausible value was used, e.g., at basal state, hepatic clearance accounted for 60% of the total clearance, the new model provided a fit similar to that of the original model (data not shown). However, values of basal insulin clearance were plausible in only 17 of our 20 subjects (in 3 subjects, implausibly high values of basal clearance of 5.8, 4.2, and 3.4 l/min were estimated, associated with elevated values of secretion indexes). In the remaining 17 subjects, the new model provided an average insulin clearance that was very similar and well correlated to that estimated with the original model (1.7 ± 0.5 vs. 1.6 ± 0.3 l/min, R = 0.74, P = 0.001). Overall, HE indexes were also very similar and well correlated to those derived from the original model (HEb = 69 ± 9 vs. 72 ± 7%, R = 0.71, P = 0.001; HE = 57 ± 12 vs. 58 ± 10%, R = 0.72, P < 0.001), thus indicating that HE indexes of the original model are robust with respect to variations in insulin clearance observed during an IVGTT.

**HE and C-peptide/insulin molar ratios.** Finally, let us relate our HE indexes to C-peptide/insulin molar ratios, which are often used as indicators of hepatic insulin extraction. In Polonsky and Rubenstein (14), the use of the ratio between incremental areas under plasma C-peptide and insulin concentration curves as a reflection of HE was discussed, and it was pointed out that the marked difference in the plasma half-lives of insulin and C-peptide complicates the interpretation of changes in this ratio. By integrating Eq. 9, it is possible to link the incremental area under insulin concentration to the total amount of delivered insulin and to insulin clearance rate (nV1)

\[
\int_0^T I(t)\, dt = \frac{V_1}{n \cdot V_1} \int_0^T \text{idr}(t)\, dt + \int_0^T \text{U}(t)\, dt
\]

where the integral of the exogenous input U(t) is the administered insulin dose.
Similarly, by integrating Eq. 1, one has a relationship between the incremental area under C-peptide concentration, the total amount of secreted hormone, and C-peptide clearance rate ($k_0V_C$):

$$\int_0^T CP(t)dt = \frac{V_C}{k_0V_C} \int_0^T idr(t)dt \quad (19)$$

During an IM-IVGTT, the ratio between incremental C-peptide and insulin areas (Eqs. 18 and 19) represents only a semiquantitative indicator of HE, because it also depends on the total amount of infused insulin and insulin and C-peptide clearance rates. Conversely, the minimal-model index HE (Eq. 16) represents a correct measure of HE in terms of the incremental areas of pancreatic and posthepatic secretion.

In conclusion, insulin secretion, kinetics, and hepatic extraction can be assessed by using the insulin and C-peptide minimal models on IM-IVGTT data. These models, used in conjunction with the minimal model of glucose kinetics (3), thus provide indexes such as insulin sensitivity, glucose effectiveness, first- and second-phase responsivity to glucose of insulin secretion, insulin clearance, and hepatic extraction, which represent a rich parametric picture of the glucose-regulatory system that is usable for studying pathophysiological states such as impaired glucose tolerance, obesity, and diabetes.

**APPENDIX**

To include in the model a link between insulin clearance and HE, insulin clearance was described as the sum of two processes: hepatic clearance, varying in time with a pattern proportional to hepatic extraction; and peripheral clearance, which was assumed to be constant. Thus the equation describing insulin kinetics is

$$I(t) = -[n^sHE(t) + n^p]I(t) + idr(t) + U(t)V_1 \quad I(0) = 0 \quad (A1)$$

where $n^s HE(t)$ (min$^{-1}$) represents hepatic clearance per unit distribution volume, modulated by HE, and $n^p$ (min$^{-1}$) represents the contribution of peripheral clearance, which was assumed to be constant during the experiment.

For HE, Eq. 15 was used

$$I(t) = -\left[n^s \left(1 - \frac{IDR(t)}{ISR(t)} \right) + n^p \right]I(t) + idr(t) + U(t)V_1 \quad I(0) = 0 \quad (A2)$$

where ISR is described by Eq. 5, whereas IDR now accounts for both hepatic and peripheral clearance:

$$IDR(t) = [dr(t) + idr(t)]V_1 = [n^sHE(t) + n^p]b + idr(t)]V_1 \quad (A3)$$

The new insulin model consists of Eq. A2 coupled with Eq. 10 to describe idr. It was simultaneously identified with the C-peptide model, Eqs. 1–4, which provides a description for ISR appearing in Eq. A2. To achieve numerical identification, a constraint was introduced on the relative contribution of hepatic and peripheral clearance in the basal state; namely, it was assumed that, in all subjects, hepatic and peripheral clearance account for 60 and 40%, respectively, of the basal clearance:

$$\frac{n^sHE}{n^p} = \frac{60}{40} \quad (A4)$$

**REFERENCES**


